M.Sc. 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Real Analysis)

Paper MTM – 101

FULL MARKS : 50 :: Time : 2 hours

Group-A

Answers any four questions out of eight questions: $4\times2 = 8$

1. a) Show that any finite subset of $\mathbb{R}$ is compact.

   b) Show that if a Cauchy sequence in a metric space has a convergent subsequence, then the whole sequence is convergent.

   c) Is the following a connected subset of $\mathbb{R}^2$ $\{(x,y) \in \mathbb{R}^2 : x^2y^2 = 1\}$? Justify your answer.

   d) Evaluate the Lebesgue integral $\int_0^5 f(x)\,dx$ where

   $$f(x) = \begin{cases} 0, & 0 \le x < 1 \\ 1, & 1 \le x < 2, 3 \le x < 4 \\ 2, & 2 \le x < 3, 4 \le x \le 5. \end{cases}$$

   e) Show that subtraction of two complex measurable functions on a measurable set X is measurable.

   f) Define σ −algebra with an example.

   g) Show that the function f defined by the $f(x) = x^{\frac{1}{3}},\ x \in [0,1]$ is a function of bounded variation although f' is not bounded on [0, 1].

   h) Let $A = \{0\}$ and $B = (0,1)$. Check whether A and B are separated in the metric spaces ($\mathbb{R}$, usual metric) and ($\mathbb{R}$, discrete metric) respectively.

Group-B

Answers any four questions out of eight questions: $4\times4 = 16$

2. a) Show that if a subset K of $\mathbb{R}$ is limit point compact then K is sequentially compact.

b) Establish a necessary and sufficient condition for a function $f:[a,b] \to \mathbb{R}$ to be a function of bounded variation on $[a,b]$.

c) Let $f(x) = x[x], x \in [1,3]$. Show that f is a function of bounded variation on $[1,3]$. Find the variation function of f on $[1,3]$ and express f as a difference of two monotonic increasing function on $[1,3]$.

d) Suppose f is bounded on $[a,b]$, f has finitely many points of discontinuity on $[a,b]$ and α is continuous at every point at which f is discontinuous. Then show that $f \in \Re(\alpha)$ on $[a,b]$.

e) Evaluate the Riemann-Stieltjes integrals:

$i)$ $\int_0^1 x\, dx^2$, ii) $\int_0^\pi x\, d\, cos x.$

f) If in a metric space (X, d) the distance between two sets A and B is a positive real then prove that the sets are separated.

g) Let $f_n: X \to \mathbb{R}^*$ be measurable for $n = 1, 2.3, ...$Then show that $\liminf\limits_{n\to\infty} f_n$ and $\inf\limits_{n\to\infty} f_n$ are measurable functions on X.

h) Let f be a measurable function on E. Then show that f^+ and f^- are integrable over E iff $|f|$ is integrable over E.

Group-C

Answers any two questions out of four questions: $2{\times}8 = 16$

3. a) Show that if f is a measurable function on E and $f = g$ almost everywhere on E, then g is measurable on E.
   a) State Lebesgue Monotone Converges theorem.
   b) In the case of Fatou's lemma if we drop the non negativeness of the sequences then the lemma does not hold. Give an example for it and explain. 3+2+3

4. a) Let $f(x) = \frac{1}{x^p}$ if $0 < x \leq 1$ and $f(0) = 0$. Find necessary and sufficient condition on p such that $f \in L^1[0,1]$. Compute $\int_0^1 f(x)\lambda(x)$ in that case.
   b) Evaluate the following: $\int_{-1}^{3} 2\, cos\ x\ d(2x + [x])$. 6+2
5. a) Show that a metric space X is compact if and only if every real valued continuous function in bounded on X.
   b) Show that every path connected metric space is connected. Give an example to show that the converse is not true. 4+4

6. a) Show that the outer measure of the Cantor set is zero.
   b) Evaluate the R-S integral:
$$\int_0^{\pi}(x+1)\ d(\sin x + cosx).$$
5+3

[Internal Assessment -10]

M.Sc. 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Complex Analysis)

Paper MTM – 102

FULL MARKS : 50 :: Time : 2 hours

Group-A

Answers any four questions out of eight questions: $4\times 2 = 8$

1. a) Sketch $S = \{z: \left|\frac{z+1}{z-1}\right| < 1\}$ and decide whether it is domain.

   b) Find the value of x and y for which $\sin(x+iy) = \cosh(4)$.

   c) Show that the function$f(z) = \frac{z}{e^z-1}$ has a removable singularity at the origin.

   d) Evaluate: $\int_{|z|=1} z\bar{z}\ dz$.

   e) Find the coefficient of $\frac{1}{z^3}$ in the Laurent series expansion of $f(z)$ for $z > 2$ where$f(z) = \frac{1}{z^2-3z+2}$.

   f) Find the Mobious transformation that maps $0, 1, \infty$ to the respective points $0, i, \infty$.

   g) State the Rouches theorem.

   h) Is the function $u = 2xy + 3xy^2 - 2y^3$ is harmonic?

Group-B

Answers any four questions out of eight questions: $4\times4 = 16$

2. a) Suppose that $f(z) = u(x,y) + iv(x,y)$ and $f'(z)$ exists at a point $z_0 = x_0 + iy_0$. Then prove that the first order partial derivatives of u and v must exists at (x_0, y_0) and they must satisfy the Cauchy Riemann equations:$u_x = v_y$; $u_y = -v_x$ at (x_0, y_0). Also prove that $f'(z_0)$ =$u_x + iv_x$ at (x_0, y_0).

b) Evaluate the integral $\int_C \frac{f(z)+f(-1/z)}{(z-i)^2}\, dz$ where C is the simple closed contour $|z - i| = \frac{1}{2}$, in counter clockwise sense and $f(z)$ is analytic in $|z - i| \leq 1$.

c) Define Meromorphic function. If $f(z) = u(x,y) + iv(x,y)$ and $g(z) = v(x,y) + ik(x,y)$ be non-zero analytic functions on $|z| < 1$. Then prove that $f(z)$ is a constant function.

d) Find the real part of the analytic function whose imaginary part is $x^2 - y^2 + \frac{x}{x^2+y^2}$. Also find the analytic function.

e) Using the method of residues: Evaluate $\int_0^{2\pi} \frac{d\phi}{a+\cos\phi}, a > 1$.

f) Define zero's of an analytic function. Find the zero's of the function $f(z) = coshz$.

g) State and prove the Cauchy's Residue Theorem.

h) State and prove the Jordon's Lemma.

Group-C

Answers any two questions out of four questions: $2\times 8 = 16$

3. a) Classify the singularity $z = 0$ of the function $f(z) = \frac{\cosh(z^3-1)}{z^7}$ in terms of removable, pole and essential singularity. In case $z = 0$ is a pole, specify the order of the pole.

   b) Evaluate the residue of the function $f(z) = \frac{\cosh(z^3-1)}{z^7}$ at $z = 0$.

   c) Using part (b) $f(z) = \frac{\cosh(z^3-1)}{z^7}$, where $C: |z| = 1$ taken in the positive direction. 4+2+2

4. a) Define Laurent's series. Find the Laurent's series expansion of $\sin\frac{1}{z}$.

   b) Evaluate : $\int_0^\infty \frac{\cos mx}{x^2+a^2}\, dx, m > 0.$ 3+5

5. a) Use the Schwartz-Chritoffel transformation to arrive at the transformation $w = z^m\,(0 < z < 1)$, which maps the half plane $y \geq 0$ onto the wedge $|w| \geq 0, 0 \leq \arg w \leq m\pi$ and transforms the point $z = 1$ to the point $w = 1$.

   b) Let $w = f(z) = \frac{\mathrm{az+b}}{cz+d}$ is a bilinear transformation. Then find the inverse of the transformation. Is it again a bilinear? Also find the determinant of both the transformations. 6+2

6. State Cauchy's Integral Theorem. Using the method of residue calculus, Evaluate: $\int_0^\infty \frac{\sin x}{x^p}\, dx, 0 < p < 1.$ 2+6

[Internal Assessment -10]

M.Sc 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Ordinary Differential Equations and Special Functions)

Paper MTM – 103

FULL MARKS : 50 :: Time : 2 hours

1. Answer any four questions $2\times4=8$

(a) Find all the singularities of the following differential equation and then classify:
$z^2(z^2-1)^2\omega''-z(1-z)\omega'+2\omega=0$

(b) Let $y_1(x)$ and $y_2(x)$ be two linearly independent solution of
$x^2\ddot{y}-2x\dot{y}-4y=0$,forall x in [0,10]
consider the Wronskian $W(x)=y_1(x)\,y_2{}'(x)-y_1{}'(x)\,y_2(x)$
If $W(1)=1$ then find the value of $W(3)-W(2)$?

(c) What are Bessel's functions of order n? State for what values of n the solutions are independent of Bessel's equation of order n.

(d) Define fundamental set of solutions and fundamental matrix for system of differential equations.

(e) When a boundary problem is a Sturm-Liouville problem**.**

(f) Prove that F(-n, b, b, -z)=$(1+z)^2$ where F(a, b, c, z) denotes the hypergeometric function.

(g) Under suitable transformation to be considered by you , prove that Legendre differential equation can be reduced to hypergeometric equation.

(h) Consider the boundary value problem

$$\frac{d^2y}{dx^2}+\lambda y=0,0\le x\le\pi$$

Subject to y(0)=0,$y(\pi)=0$. *Find the values of* λ for which the boundary value problem is solvable.

2 **Answer any four questions** $4\times4=16$

(a) Prove that $\int_{-1}^{1} P^2{}_n(z)dz = \frac{2}{2n+1}$. where $P_n(z)$ is the Legendre's Polynomial of degree n.

(b) Deduce Rodriguies formula for Legendre's Polynomial. 4

(c) Prove that for the hypergeometric function

$$F(\alpha,\beta,\gamma,z) = \frac{\Gamma(\gamma)}{\Gamma(\beta)\Gamma(\gamma-\beta)}\int_0^1 t^{\beta-1}(1-t)^{\gamma-\beta-1}(1-zt)^{-\alpha}\,dt.$$

(d) Using Green's function method ,slove the following differential equation $\boldsymbol{y'''(x) = 1}$

Subject to boundary $conditions$ $y(0) = y(1) = y'(0) = y'(1)$.

(e) Show that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of

$$\exp\left(\frac{x(z-\frac{1}{z})}{2}\right).$$

(f) Prove that for the Bessel's function $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

(g) show that $1 + 3P_1(z) + 5P_2(z) + 7P_3(z) + \cdots + (2n+1)P_n(z) = \frac{d}{dz}\,[P_{n+1} + P_n(z)]$

Where $P_n(z) denotes\ the\ Legendre's$ polynomial of degree n

(h) If the vector functions $\phi_{1,}\phi_{2,}\cdots,\phi_n$ defind as follows

$$\phi_1 = \begin{bmatrix}\phi_{11}\\ \phi_{21}\\ \vdots\\ \phi_{n1}\end{bmatrix},\ \phi_2 = \begin{bmatrix}\phi_{12}\\ \phi_{22}\\ \vdots\\ \phi_{n2}\end{bmatrix},\cdots\ \phi_n = \begin{bmatrix}\phi_{1n}\\ \phi_{2n}\\ \vdots\\ \phi_{nn}\end{bmatrix}$$

be n solution of the homogeneous linear differential equation $\frac{dx}{dt} = \mathrm{A(t)x(t)}$ in the interval $a \le t \le b$, then these n solutions are linearly independent in $a \le t \le b\ iff$ Wronskian $\mathrm{W}[\phi_{1,}\phi_{2,}\cdots,\phi_n] \ne 0\ \forall\ t\ on\ \ a \le t \le b.$

3 **Answer any two questions** $8\times 2=16$

(a) (i) Show that all the eigen values of the regular Sturm-Liouville system $\frac{d}{dx}[p(x)\frac{dy}{dx}]+[q(x)+\lambda r(x)]y=0$ are real with respect to weight function r(x)>0 **5**

(ii)Find the values of a and b for which the boundary value problem $x^2\ddot{y}-2x\dot{y}+4y=0$ subject to the boundary conditions $y(1)+ay'(1)=1$ and $y(2)+ay'(2)=2$ has a unique solution. 3

(b) (i)Discuss Frobenious method of finding the series solution about the regular singular point at the origin for an ODE of 2^{nd} order when the roots of the indicial equation are unequal and not differ by an integer.. 6

(ii) Define Green's function of the differential operator L of the non-homogeneous differential equation: Lf(x)=f(x). 2

(c) (i) Find the characteristics values and characteristic functions of the Sturm-Liouville Problem $(x^3y')'+\lambda xy=0; y(1)=0, y(e)=0.$ 4

(ii) Determine whether the matrix B= $\begin{pmatrix} e^{4t} & 0 & 2e^{4t} \\ 2e^{4t} & 3e^{t} & 4e^{4t} \\ e^{4t} & e^{t} & 2e^{4t} \end{pmatrix}$ is a fundamental matrix of the system $\frac{dX}{dt}=AX$ where X= $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, A= $\begin{pmatrix} 1 & -3 & 9 \\ 0 & -5 & 18 \\ 0 & -3 & 10 \end{pmatrix}$ .If not Justify and find another alternative fundamental matrix. 4

(d) (i) Prove that if $f(z)$ is continuous and has continuous derivatives in $[-1,1]$

Then $f(z)$ has unique Legendre series expansion is given by

$f(z)= \sum_{n=0}^{\infty} c_n P_n(z)$ where P_n' $s\ are\ Legendre\ polynomials$ 6

$c_n=\frac{2n+1}{2}\int_{-1}^{1} f(z)P_n(z)dz, n=1,2,3\ \ldots\ldots$

(ii) Prove that $\frac{d}{dz}[J_0(z)]=-J_1(z)$. 2

[Internal Assessment : 10]

M.Sc 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

Advanced Programming in C and MATLAB

Paper MTM – 104

FULL MARKS : 50

Time : 2 hours

1. Answer any four questions 4×2=8
   - (i) Write a program in MATLAB to find the sum of integers and fractional parts of a series of numbers.
   - (ii) What is right division ? Explain with example.
   - (iii) Write a script in MATLAB to find the roots of the polynomial $3x^3 - 2x + 6$.
   - (iv) What is the cell array ? how will you write the name,address and age of three students in a cell array in MATLAB?
   - (v) Summarize the rules for assigning numerical values to enumeration constants. What default values are assigned to enumeration constant
   - (vi) What is **type casting** and **const** qualifier in C language ? Illustrate with example.
   - (vii) Explain the functions fopen() ,fclose().
   - (viii) Write a note on the preprocessor operators $\# \; and \; \#\#$
2. Answer any four questions. 4×4=16
   - (i) Write a program in MATLAB to add two arrays or two matrices.
   - (ii) Describe the types of variables in MATLAB according to their scopes in two or more functions.
   - (iii) $Given\ x =$ [2,7,4,8,0,10] what will be the values of y for the following relation:

     $(a)\ y = x > 5$

     (b) $y = x\ (\,find\ (x > 5))$

     (c) $y = length(\,find(x > 5))$

     (d) $y = all\big(find(x > 5)\big)$
   - (iv) what are the functions ‘deconv’ and ‘polyder’ in MATLAB ? Explain with example.
   - (iv) How is a multidimensional array defind in terms of an array of pointers? What does each pointer represent ? How elements can be accessed in this case?
   - (v) Write a programe to interchange the value of two variables using pointers .

(vii) what is meant by low –level programming ? suppose that v is a signed 16-bit integer quantity whose hexadecimal value is 0x369c. Evaluate each of the following shift expressions (utilize the original value of v in each expression.)

(a) V << 4 and (b) v >> 4

(viii) What is ment by dynamic memory allocation ? Distinguish between malloc and calloc.

3. Answer any two question

(i) what do you mean by recursive function ? write a function in MATLAB to find the value of a determinant of any order recursively . 2+6

(ii)

(a) Write a script in MATLAB to find the product of the diagonal element of a square matrix.

(b) Write a script in MATLAB to create a matrix of desired size from an array input using 'sscanf' function. 4+4

(iii) What is a self –referential structure ? For What kinds of application are self –referential structures useful? Write a program in C to construct a linked list containing three components, where each component consists of two data items :a string and a pointer that references ,the next component within the list. 1+1+6

(vi) What is difference between printf and fprintf ? Write a programe which will save a text file to another file. 2+6

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[Internal Assessment -10]

M.Sc 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Classical Mechanics and Non linear Dynamics)

Paper MTM – 105

FULL MARKS : 50

Time : 2 hours

1. Answer any four questions $2\times 4=8$

(a) State the fundamental postulates of special theory of relativity

(b) Prove that if the transformation does not depend explicity on time then the Hamiltonian represents the total energy.

(c) Prove that the Work done by the force in displacing the particle from the position P_1 to P_2 is equal to the difference between the potential energies of the particle at those two positions.

(d) Suppose a rigid body is rotating about a fixed point. Prove that the kinetic energy is conserved throughout the motion.

(e) Show that the rate of change of angular momentum is equal to the applied torque for a system of particles.

(f) Write a brief note on phase portrait.

(g) Define poisson brackets .show that it does not satisfy commutative property.

(h) Prove that $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz Transformation.

2 Answer any four questions $4\times 4=16$

(a) Construct the Routhian for the two –body problem , for which

$$L = \frac{\mu}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - V(r)$$

(b) A double pendulum consisting of two masses m_1 and m_2 oscillates in a vertical plane through small angles . m_1 is suspended from a fixed point by light inextensible string of length l_1 and $m_2$is suspended from m_1 by a similar string of length l_2 .
State the number of degrees of freedom of the system and find the equations of motion by using Lagrange's formulation.

(c) Prove that the Poisson bracket of two constants of motion is itself a constant even when the constant s depend on time explicitly

(d) Prove that the phase volume is invariant under canonical transformation..

(e) Prove that the linear autonomous plane systems $\frac{dx}{dt}=ax+by, \frac{dy}{dt}=cx+dy$ is stable if $p^2-4q>0$, q>0 , p<0.

(f) Prove that Poisson braket is invariant under canonical transformation.

(g) use Hamilton's equation to find the equation of motion of a simple pendulum.

(h) State and prove the Jacobi Identity .

3. Answer any two questions 8×2=16

(a) (i) A body moves about a point O under no forces . The principal moments of inertia at O being 3A,5A and 6A. Initially ,the angular velocity has components $\omega_1 = n, \omega_2 = 0, \omega_3 = n$ about the corresponding principal axes .Show that at any time t ,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tanh(\frac{nt}{\sqrt{5}})$$

And that the body ultimately rotates about the mean axis.

(ii) If a transformation from q,p to Q,Pbe canonical then bilinear from $\sum_i(\delta p_i dq_i - \delta q_i dp_{i)})$ remains invariant

(b) (i) Let $J = \int_{x_0}^{x_1} F(y, y', x)dx$, where y is an unknown function depends on x .Derive a differential equation to find the curve $y = y(x)$ which will minimize J.

(ii) Derive Lagrange's equations of motion from the Hamilton's principle.

(c) Consider the equilibrium configuration of the molecule such that two of its atoms of each of mass M are symmetrically placed on each side of the third atom of mass m. All three atoms are collinear. Assume the motion along the modecules and there being no interaction between the atoms. Compute the kinetic energy and potential energy of the system and discuss the motion of the atoms. 8

(d) Prove that E= MC^2 in relative mechanics.

[Internal Assessment -10]

M.Sc. 1st Semester examination, 2018

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Graph Theory)

Paper MTM – 106

FULL MARKS : 25 :: Time : 1 hours

Group-A

Answers any two questions out of four questions: $2\times2 = 4$

1. a) What is the maximum number of vertices in a graph with 35 edges and all vertices are of degree at least 3.

   b) Prove that there is one and only one path between every pair of vertices in a tree T.

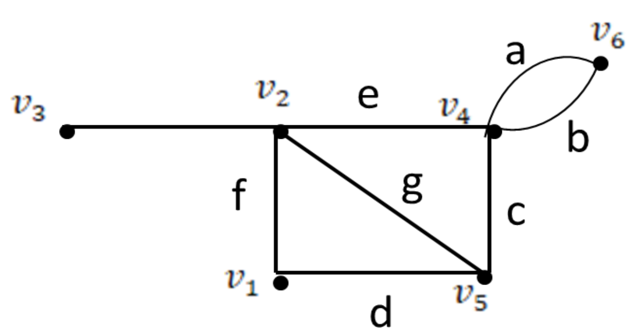
   c) Define Euler graph. Find the value of m and n for which $K_{m,n}$ be the Euler graph. 1+1

   d) Explain spanning tree of a connected graph.

Group-B

Answers any two questions out of four questions: $2\times4 = 8$

2. a) What is Chromatic Polynomial of a graph? Show that Chromatic Polynomial of a tree with n vertices is $P_n(\lambda) = \lambda(\lambda - 1)^{n-1}$.
   b)



i) Find path matrix between (v_3, v_4),
ii) Find the circuit matrix.

c) Define binary tree. Find the number of pendent vertices in a binary tree with n vertices.

d) Define separable graph. Show that vertex connectivity of a graph G can never exceed the edge connectivity of G.

Group-C

3. Answers any one questions out of two questions: $1{\times}8 = 8$

a) i) Prove that a tree with n vertices has $n - 1$ edges.

ii) What is the maximum number of vertices in a graph with 30 edges and all vertices are of degree at least 3.

iii) Prove that if G is self complementary then G has $4k$ or $4k + 1$ vertices, where k is an integer. 4+2+2

b) Prove that with respect to a given spanning tree T, a branch b_i that determines a fundamental cut-set S is contained in every fundamental circuit associated with the chords in S and in no others. Show that any simple connected planner graphs satisfy the inequality $e \leq 3n - 6$ where n and e are the number of vertices and edges of the graph respectively.

4+4

[Internal Assessment -5]